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Carsten Lunde Petersen and Gustav Ryd

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Convergence of rational rays in parameter spaces

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Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c\in\mathbb{C}}$, $d \geq 2$.

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Convergence of rational rays in parameter spaces

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Abstract

We give an elementary proof of the landing Theorem for rational external rays of the Mandelbrot set and related connectedness loci for the one-parameter families of polynomials $\{P_c(z) = z^d + c\}_{c\in\mathbb{C}}$, $d \geq 2$.

Throughout this paper the integer $d \geq 2$ will be arbitrary but fixed, and used without further notice. It will be the degree of our polynomials. Let $P_c(z) = P_c = z^d + c$, $c \in \mathbb{C}$, denote the family of monic degree d polynomials with a degree $(d-1)$ critical point at the origin. For each c let J_c denote the Julia set for P_c and define the domain of attraction to infinity

$$\Lambda_c = \Lambda_c(\infty) = \{z \in \mathbb{C} | P_c^n(z) \longrightarrow \infty, \text{as } n \to \infty\}.$$

Let $\phi_c : \widetilde{\Lambda}_c \longrightarrow (\mathbb{C} \setminus \overline{\mathbb{D}})_c$ be the maximal univalent 'radial' (on the image side) extension of the unique Bötcher coordinate tangent to the identity at infinity.

Define $M_d = \{c \in \mathbb{C} | c \notin \Lambda_c\}$ and let $\Phi_d : \overline{\mathbb{C}} \setminus M_d \longrightarrow \overline{\mathbb{C}} \setminus \overline{\mathbb{D}}$ denote the unique Riemann map tangent to the identity at infinity, then $\Phi_d(c) = \phi_c(c)$. Given $\theta \in \mathbb{T} = \mathbb{R}/\mathbb{Z}$ and $c \in \mathbb{C}$: The dynamical (external) ray $R_c(\theta)$ of J_c is the analytic arc $\phi_c^{-1}(\{\exp(s + i2\pi\theta) | s > 0\} \cap (\overline{\mathbb{C}} \setminus \overline{\mathbb{D}})_c)$. It starts at ∞ and ends either at a precritical point $z_0 \in \cup_{n\geq 0} P_c^{-n}(c)$ or on the Julia set J_c. The parameter (external) ray $R_{M_d}(\theta)$ of M_d is the analytic arc $\Phi_d^{-1}(\{\exp(s + i2\pi\theta) | s > 0\})$. A ray is called a rational ray if θ is rational, i.e. $\theta \in \mathbb{Q}/\mathbb{Z}$. A ray R is said to land or converge if $\overline{R} \setminus R$ is a singleton subset of J (if it is a dynamical ray) or M_d (if it is a parameter ray).

On the boundary of M_d we distinguish two particular and different types of parameters, parabolic parameters and Misiurewicz parameters. A parameter c_0 is called parabolic, if P_{c_0} has a parabolic cycle, i.e., there exists a positive integer n and a periodic point p such that $P_{c_0}^n(p) = p$ and $(P_{c_0}^n)'(p) = 1$.

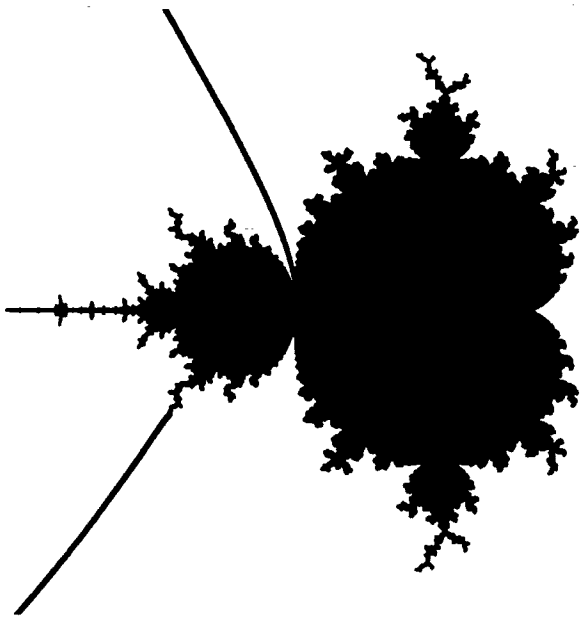

Figure 1: The Mandelbrot set with a periodic and a strictly preperiodic ray.

A parameter c_0 is called a Misiurewicz parameter, if the orbit of c_0 is finite and ends in a repelling periodic orbit.

In this paper we give an elementary proof of the landing Theorem for rational external rays of M_d, for any integer $d \geq 2$:

Theorem 1 *Given $\theta \in \mathbb{T}$ with $d^l\theta \equiv d^{l+q}\theta \mod 1$, for some minimal integers $l \geq 0$ and $q \geq 1$. Then the parameter ray $R_{M_d}(\theta)$ lands on a parameter $c_0 \in \partial M_d$. Furthermore,*

1. *if $l = 0$ then c_0 is a parabolic parameter, the (unique) parabolic orbit for P_{c_0} has one cycle of immediate attracted basins. This cycle contains both 0 and c and its exact period is q. Moreover the dynamic ray $R_{c_0}(\theta)$ lands on the parabolic point to which c_0 is attracted under iteration by $P_{c_0}^q$.*

2. *if $l > 0$ then c_0 is a Misiurewicz parameter, $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$ and the dynamic ray $R_{c_0}(\theta)$ land on c_0.*

This Theorem is well-known, at least for $d = 2$, since [DH]. The original proof is however rather involved. There are other proofs, such as [HS], which

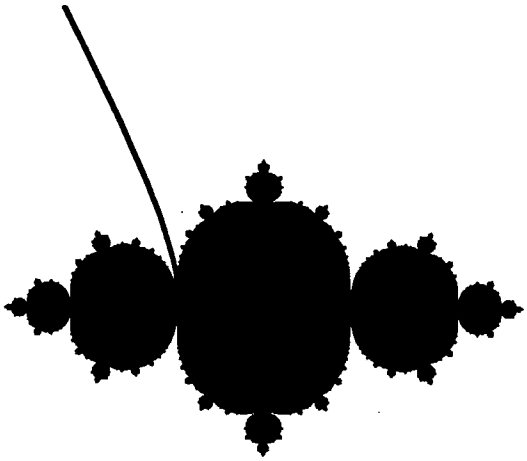

Figure 2: The Julia set for a quadratic polynomial with a period-2 ray landing at a parabolic fixed point.

uses iteration on Teichmüller spaces, and [Mil], [Sch], which both depend on global counting arguments and a priori analysis of the sets M_d. In [CG] this theorem is claimed, but there seems to be a part missing. This paper has evolved from [Pet] and [Ryd], which both were extending Theorem 1 to different settings but using similar techniques.

Our proof differs from the original proof in [DH] mainly in our direct approach to convergence and uses only elementary analytical means. Moreover it has easy generalizations to many other settings. We want to make our proof available to the public, because we believe that our elementary approach will be of use also to others.

We will frequently use $g_c : \mathbb{C} \longrightarrow \mathbb{R}_+$, the Green's function for J_c with pole at ∞, which is the subharmonic function which is harmonic on Λ_c, coincides with $\log|\phi_c(z)|$ on $\widetilde{\Lambda}_c$ and which is identically zero on $\mathbb{C} \setminus \Lambda_c$. Similarly, the Green's function for M_d with pole at ∞, $G_{M_d} : \mathbb{C} \longrightarrow \mathbb{R}_+$ is the subharmonic function, which is 0 on M_d and which coincides with $\log|\Phi_{M_d}|$ on $\mathbb{C} \setminus M_d$. Note that an external ray is a gradient line for the appropriate Green's function.

We first prove that the end of the external ray gives rise to a sequence of

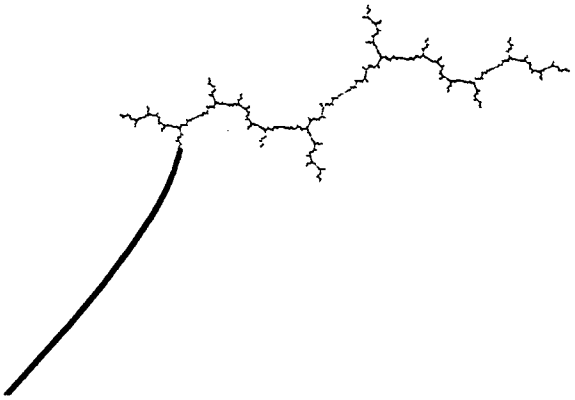

Figure 3: The Julia set for a quadratic polynomial with a preperiodic ray landing at a Misiurewicz point.

limit maps which relates the dynamics in the filled Julia set, K_{c_0}, with the dynamics in Λ_{c_0}.

Given $c_0 \in \mathbb{C}$ and $\phi \in \mathbb{T}$, for $0 < \epsilon$ and $w \in R_c(\phi)$ we define a 'rectangle'

$$V(w,\epsilon) = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^q g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\}.$$

Proposition 1.1 *Given $\theta \in \mathbb{T}$ with $d^l\theta \equiv d^{l+q}\theta \mod 1$, for some minimal $l \geq 0$ and $q \geq 1$. Let $c_0 \in \partial M_d$ be a limit point of the ray $R_{M_d}(\theta)$. Then there exists $w \in R_{c_0}(d^l\theta)$, $\epsilon > 0$ and a sequence of maps $\{\psi_n : V \longrightarrow K_{c_0}\}_{n\geq 0}$, $V = V(w,\epsilon)$ such that*

1. *For all $0 \leq n$: $\psi_n(w) = P^n_{c_0}(c_0)$ and $\psi_{n+1} = P_{c_0} \circ \psi_n$.*
2. *For all $l \leq n$: $\psi_n \circ P^q_{c_0} = P^q_{c_0} \circ \psi_n$, wherever defined and in particular $\psi_n(P^q_{c_0}(w)) = P^{n+q}_{c_0}(c_0) = \psi_{n+q}(w)$.*
3. *One of the following mutually exclusive cases occur*
   (a) *The ψ_n are univalent and for all $0 \leq n \neq m$ with either $n < l$ or $|n - m| \neq q$: $\psi_n(V) \cap \psi_m(V) = \emptyset$.*

(b) All ψ_n are constants.

Proof : Let $\phi = d^l\theta \mod 1$ be the first periodic angle in the orbit of θ. And let $\{c_j\}_{j\in\mathbb{N}} \subset R_{M_d}(\theta) \cap \{c|G_{M_d}(c) < d^{-l}\}$ be a sequence converging to c_0. For each j define N_j by $P_{c_j}^{N_j}(c_j) := w_j \in R_{c_j}(\phi) \cap \{z|1 \le g_{c_j}(z) < d^q\}$. Passing to a subsequence if necessary, we can suppose by relative compactness that $P_{c_j}^{N_j}(c_j) \longrightarrow w \in R_{c_0}(\phi)$ with $1 \le g_{c_0}(w) \le d^q$.

Fix $0 < \epsilon < 1/2$ and define a simply connected domain $V = V(w, \epsilon)$. We can assume that $w_j \in V$. For each j and $0 \le n \le N_j$ let $\psi_{n,j} : V \longrightarrow \mathbb{C}$ be the unique branch of $P_{c_j}^{-N_j+n}$ which maps w_j to $P_{c_j}^n(c_j)$. Then for each fixed

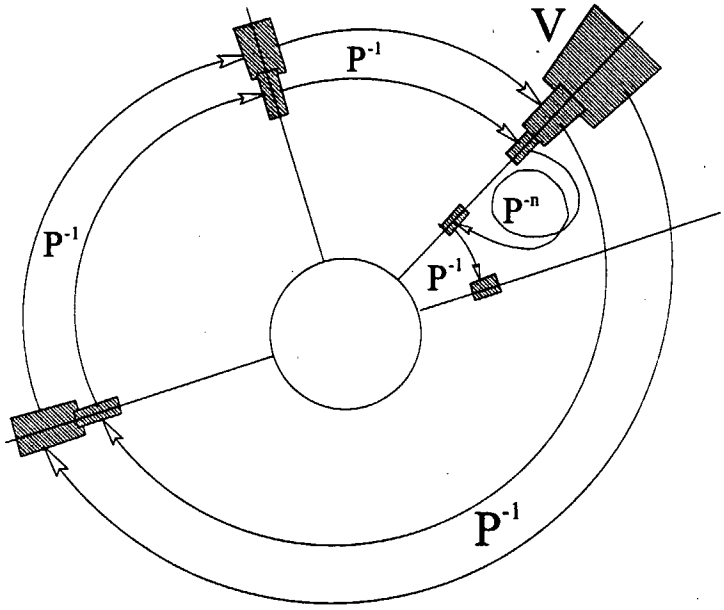

Figure 4: The domain V and the image domains $\psi_{n,j}(V)$ viewed in the Bötcher coordinate at infinity.

j the uniformly bounded univalent maps $\psi_{n,j}$ satisfy

$$\forall\, 0 \le n \le N_j : \qquad \psi_{n,j}(w_j) = P_{c_j}^n(c_j) \quad \text{and} \quad \psi_{n+1,j} = P_{c_j} \circ \psi_{n,j} \tag{1.1}$$

$$\forall\, l \le n \le N_j - q : \qquad \psi_{n,j} \circ P_{c_j}^q = P_{c_j}^q \circ \psi_{n,j}. \tag{1.2}$$

$$\forall\, 0 \le n \ne m \le N_j \text{ with } n < l \text{ or } |n-m| \ne q : \qquad \psi_{n,j}(V) \cap \psi_{m,j}(V) = \emptyset \tag{1.3}$$

Here the disjointness property (1.3) holds if ϵ is small enough. Note that $N_j \longrightarrow \infty$ as $j \to \infty$. As the univalent maps $\psi_{n,j}$ are uniformly bounded on V, we can suppose, using the Cantor diagonal principle to extract a subsequence if necessary, that for each $n \in \mathbb{N}$

$$\psi_{n,j} \underset{j\to\infty}{\rightrightarrows} \psi_n$$

where $\psi_n : V \longrightarrow \mathbb{C}$ is some holomorphic map, which is either univalent or constant by Hurwitz Theorem. Note that $g_{c_0} = 0$ on $\psi_n(V)$ so $\psi_n(V) \subset K_{c_0}$. By uniform convergence the properties (1.1), (1.2) and (1.3) also holds for the ψ_n (with P_{c_0}) so Properties 1. and 2. follows. By (1.1) if one ψ_n is constant they all are, so Property 3. follows. **q.e.d.**

Proposition 1.2 *Let θ and c_0 be as in Proposition 1.1. If $c_0 \in J_{c_0}$ then 3b. holds, $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$, $l > 0$ and the dynamic ray $R_{c_0}(\theta)$ lands on c_0.*

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. The image $\psi_n(V) \subset K_{c_0}$. Thus if $c_0 = \psi_0(w) \in J_{c_0} = \partial K_{c_0}$ then ψ_0 cannot be open thus all ψ_n are constant. Combining Properties 1. and 2. we have $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$. If $l = 0$ then c_0 is q-periodic, but then also the critical point 0 is q-periodic, because it is the only preimage of c_0. This contradicts however the fact that $c_0 \in \partial M_d$. Hence $l > 0$ and c_0 is a Misiurewicz parameter with $P^l_{c_0}(c_0) = P^{l+q}_{c_0}(c_0)$.

To complete the proof we show that the ray $R_{c_0}(\theta)$ lands on c_0. To this end let $\{c_j\}_{j\in\mathbb{N}} \subset R_{M_d}(\theta)$ be a sequence converging to c_0 as in the proof of Proposition 1.1. Define $R^*_{c_j}(\theta) = ((R_{c_j}(\theta) \cap \{z | g_{c_j}(z) \geq g_{c_j}(c_j)\}) \cup \infty)$. Then the sequence $\{R^*_{c_j}\}_{j\in\mathbb{N}}$ is a sequence of compact sets in $\overline{\mathbb{C}}$. Thus passing to a subsequence, if necessary, we can suppose the sequence converges to some compact subset $R \subset \overline{\mathbb{C}}$ containing c_0 and ∞. Moreover $R \cap \Lambda_{c_0} = R_{c_0}(\theta)$ by continuity with respect to the parameter of the Bötcher coordinates ϕ_c. Let $z \in R \cap J_{c_0}$ be arbitrary (note that $J_{c_0} = \partial\Lambda_{c_0} = \mathbb{C} \setminus \Lambda_{c_0}$) and let $z_j \in R^*_{c_j}(\theta)$ be a sequence with $z_j \longrightarrow z$ as $j \to \infty$. Arguing as in the proof of Proposition 1.1 we find a point $\widehat{w} \in R_{c_0}(d^l\theta)$, an open set $\widehat{V}$ and a sequence of holomorphic maps $\widehat{\psi}_n : \widehat{V} \longrightarrow \mathbb{C}$ satisfying the properties 1., 2. and 3. of Proposition 1.1, except that c_0 is replaced by z. As above case 3b. in 3. holds, because $z \in J_{c_0}$. But then $P^l_{c_0}(z) = P^{l+q}_{c_0}(z)$. The set $\{z | P^l_{c_0}(z) = P^{l+q}_{c_0}(z)\}$ is however finite and hence $R \cap J_{c_0}$ is a singleton containing c_0. **q.e.d.**

Proposition 1.3 *Let θ and c_0 be as in Proposition 1.1. If c_0 belongs to a Fatou component U for P_{c_0}, then 3a. holds. Furthermore, there exists a sequence of Jordan arcs $\gamma_n : [0,1] \longrightarrow P_{c_0}^n(U)$ satisfying*

1'. For all $0 \leq n$: $\gamma_n(0) = P_{c_0}^n(c_0)$ and $\gamma_{n+1} = P_{c_0} \circ \gamma_n$.

2'. For all $l \leq n$: $\gamma_n \cap \gamma_{n+q} = \gamma_n(1) = \gamma_{n+q}(0) = P_{c_0}^{n+q}(0)$.

3'. For all $0 \leq n \neq m$ with either $n < l$ or $|n-m| \neq q$: $\gamma_n \cap \gamma_m = \emptyset$.

Proof : Let V and ψ_n be as in the conclusion of Proposition 1.1. If the ψ_n were constants then by combining Property 1. and Property 2., $P_{c_0}^l(c_0) = P_{c_0}^{l+q}(c_0)$. Hence the unique critical point is preperiodic, but then it either belongs to the Julia set or is superattracting. The first possibility is excluded by hypothesis and in the second case c_0 belongs to the interior of M_d, which is impossible. Thus ψ_l is not constant so Property 3a. holds.

Let w be as in the proof of Proposition 1.1 and define an arc $\Gamma \subset V$ by

$$\Gamma = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^q g_{c_0}(w)\}.$$

The arc Γ is naturally parmetrised linearly in the potential by the interval $[0,1]$. Define arcs $\gamma_n = \psi_n \circ \Gamma : [0,1] \longrightarrow P_{c_0}^n(U)$.

Properties 1'. and 3'. follows immediately from Properties 1. and 3a. To see also Property 2'. define

$$\widehat{V} = \{\exp(t + i2\pi\eta) | g_{c_0}(w) - \epsilon < t < d^{2q} g_{c_0}(w) + \epsilon \text{ and } |\eta - \phi| < \epsilon\},$$
$$\widehat{\Gamma} = R_{c_0}(\phi) \cap \{z | g_{c_0}(w) \leq g_{c_0}(z) \leq d^{2q} g_{c_0}(w)\}$$

and note that the ψ_n have univalent extensions $\widehat{\psi}_n$ to $\widehat{V}$. Moreover for $n \geq l$: $\widehat{\psi}_n(\widehat{\Gamma}) = \gamma_n \cup \gamma_{n+q}$ is an arc. Thus also the interior disjointness $\gamma_n \cap \gamma_{n+q} = P_{c_0}^{n+q}(c_0)$ follows. **q.e.d.**

Proposition 1.4 *Let θ, c_0 and U be as in Proposition 1.3. Then U is an immediate parabolic basin of exact period q and the preperiod $l = 0$.*

Proof : Let $U_l = P_{c_0}^l(U)$. Then $\gamma_l \in U_l$ and U_l is q-periodic, because both $P_{c_0}^l(c_0) = \gamma_l(0)$ and $P_{c_0}^{l+q}(c_0) = \gamma_l(1) \in U_l$. But then it is either a hyperbolic, a parabolic or a rotation domain, however the first implies c_0 is in the interior of M_d and the last requires at least one critical point in the Julia set and

are hence excluded. Thus U_l is an immediate parabolic basin and so is U, because the cycle of such immediate basins should contain a critical point and value. Thus U is q-periodic. Let the (exact) period of U be p. Then $\gamma_p \in U$. Let $\Phi : U \longrightarrow \mathbb{C}$ be a Fatou coordinate for $P_{c_0}^p$ on U normalized by $\Phi(c_0) = 0$ and $\Phi(P_{c_0}^p(c_0)) = 1$. The map Φ has a univalent inverse branch

$$\Psi : \{z = x + iy | y > -1\} \longrightarrow \Omega$$

with $\Psi(0) = c_0$. By compactness there exists $n \geq 0$ such that γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω. Let γ'_{pn} and $\gamma'_{p(n+1))}$ denote the corresponding images under Φ. The map $z \to \exp(2\pi ip/q)$ maps these two arcs onto two simple closed curves which are rotations (around the origin) of each other. Thus they intersect and so do γ'_{pn} and $\gamma'_{p(n+1)}$. Since γ_{pn} and $\gamma_{p(n+1)}$ are contained in Ω they also intersect. But then $p = q$ by Property 3'.

We prove that $l = 0$. Choose n such that $\gamma_{nq} \subset \Omega$ so that $\gamma_{(n+m)q} \subset \Omega$ for all $m \geq 0$. Then $\Phi(\gamma_{nq})$ satisfies the hypothesis of the isotopy Lemma 1.5 below, because of Properties 2'. and 3'. Let $\widetilde{\Gamma}$ be the corresponding isotopy of Lemma 1.5. Increasing n, if necessary, we can assume $\widetilde{\Gamma}([0,1] \times [0,1]) \subset \Phi(\Omega)$, because of Property 1'. Define $\Gamma_n : [0,1] \times [0,1] \longrightarrow \Omega$ as the isotopy $\Gamma_n = \Psi \circ \widetilde{\Gamma}$. Define isotopies $\Gamma_k : [0,1] \times [0,1] \longrightarrow U$, $n \geq k \geq 0$ recursively by Γ_{k-1} is the lift of Γ_k to $P_{c_0}^q$ with $\Gamma_{k-1}(1,[0,1]) = \Gamma_k(0,[0,1])$. An easy induction proof shows that for each $n \geq k \geq 0$ and each $0 \leq t \leq 1$: $\Gamma_k(t,0) = \Psi(k+t)$. In particular $\Gamma_k(0,[0,1]) = P_{c_0}^{kq}(c_0)$. But then $\gamma_0 \cap \gamma_q = P_{c_0}^q(c_0)$, which contradicts Property 3', when $l > 0$. **q.e.d.**

The proof of the following isotopy Lemma is left to the reader.

Lemma 1.5 *Suppose $\gamma_1 : [0,1] \longrightarrow \mathbb{C}$ is an arc with $\gamma_1(0) = 0, \gamma_1(1) = 1$ and with $\gamma_1 \cap (\gamma_1 + n) = \emptyset$ for $|n| > 1$ and with $\gamma_1 \cap (\gamma_1 + 1) = 1$, $\gamma_1 \cap (\gamma_1 - 1) = 0$. Then there exists an isotopy of arcs $\Gamma : [0,1] \times [0,1] \longrightarrow \mathbb{C}$ with $\Gamma(t,1) = \gamma_1(t)$, $\Gamma(t,0) = t := \gamma_0(t)$, $\Gamma(0,[0,1]) = 0$, $\Gamma(1,[0,1]) = 1$ and $\Gamma(]0,1[,[0,1]) \subset \mathbb{C} \setminus \mathbb{Z}$.*

Proposition 1.6 *Let θ, c_0 and U be as in Proposition 1.3. Then the dynamic ray $R_{c_0}(\theta)$ lands on the parabolic point on the boundary of U.*

Proof : Let R be as in the second part of the proof of Proposition 1.2. Let α denote the parabolic point to which c_0 is iterated by $P_{c_0}^q$. Then $\alpha \in R$ by closedness and forward invariance of R. We claim that $R \cap J_{c_0} = \alpha$. To prove this let Ω be any bounded Fatou component with $R \cap \Omega \neq \emptyset$. We shall

prove that $R \cap \partial\Omega$ is a single q-periodic parabolic point, because Ω contains at most one critical point for $P_{c_0}^q$. If the degree is 2, (the critical point is simple) there is nothing to prove, because a degree two basin with locally connected boundary contains precisely one periodic orbit with period dividing that of the basin. If the degree is higher, or one does not want to rely on local connectivity, one may procede as follows: Let $z_0 \in R \cap K_{c_0}$ be arbitrary. Redoing the proof of Proposition 1.1 but with c_0 replaced by z_0 we find $w' \in R_{c_0}(\theta)$, $V' = V'(w', \epsilon)$ and holomorphic maps $\{\psi_{z_0,n} : V' \longrightarrow K_{c_0}\}_{n \geq 0}$ satisfying the conclusions 1.–3. of that Proposition (with c_0 replaced by z_0). If $z_0 \in R \cap J_{c_0}$ then case 3.3b. occurs and z_0 is q-periodic. If z_0 belongs to the Fatou set then case 3.3a. occurs and in particular $\psi_{z_0,0} : V' \longrightarrow U$ is univalent with $\psi_{z_0,0}(w') = z_0$, $\psi_{z_0,0}(P_{c_0}^q(w')) = P_{c_0}^q(z_0)$ and

$$\psi_{z_0,0}(\{z \in R_{c_0}(\theta) | g_{c_0}(z) \geq g_{c_0}(w')\}) \subseteq R \cap U \subseteq \psi_{z_0,0}(R_{c_0}(\theta) \cap V').$$

Suppose there exists a bounded Fatou component Ω for which $L = R \cap \Omega$ connects two distinct q-periodic points α', β on the boundary of Ω. By the above any point $z \in L$ is connected to $P_{c_0}^q(z)$ in L through an analytic arc contained in L. Hence we can assume that α' is parabolic. Let $\phi : \Omega \longrightarrow \mathbb{D}$ be a Riemann map say mapping α' to 1. Let $\widehat{L} = \phi(L)$ and define $\widehat{P}$ as the conjugate Blaschke product $\widehat{P} = \phi \circ P_{c_0}^q \circ \phi^{-1}$. Then $\widehat{L}$ connects the parabolic point 1 for $\widehat{P}$ with a repelling fixed point $\widehat{\beta} \neq 1$. In fact $\widehat{L}$ is easily seen to contain an analytic arc $\widehat{\gamma}$, which connects $\widehat{\beta}$ to 1, and which is parametrized by an analytic diffeomorphism $\eta : \mathbb{R} \longrightarrow \widehat{\gamma}$ with $\eta(t+1) = \widehat{P}(\eta(t))$.

Let $U_1, U_2 \subset \mathbb{D}$ be the two complimentary components of $\mathbb{D} \setminus \eta$. Consider say U_1. It contains a unique connected component U_1' of $P^{-1}(U_1)$, because the dynamics of P on η is diffeomorphically conjugate to translation by 1. The degree of the restriction $p : U_1' \longrightarrow U_1$ is at least 2 because $\overline{U_1} \cap \mathbb{S}^1$ is covered at least twice by $\overline{U_1'} \cap \mathbb{S}^1$. Thus U_1 contains a critical point. And simillarly so do U_2. This contradicts that P has only one critical point. It follows that R contains precisely one q-periodic point. The parabolic one to which c_0 is iterated under $P_{c_0}^q$. **q.e.d.**

Remark 1.7 *Consider the family $Q_c(z) = z^4 - 2z^3 + z^2 + z + c$, $c \in \mathbb{C}$ of quartic polynomials. For the polynomial Q_0 the real axis contains one critical point $\omega \in]-\frac{1}{3}, 0[$, whose critical value v_0 is contained in the same interval and is attracted to the parabolic fixed point 0. The two other critical points are*

complex conjugate and attracted to the parabolic fixed point 1. *Moreover the external ray of argument* 0 *equals* $[1, \infty]$. *For any sequence* $\{c_n\}_{n\in\mathbb{N}} \subset]0, \infty$ *converging to* 0 *the Hausdorff limit* R *of the segments of external rays* $[v_n, \infty]$ *equals* $[v_0, \infty]$. *In this case the segment* $]0, 1[$ *is like the arc* L *in the proof of the above theorem.*

Proof of Theorem 1: The cluster set $Cl_{M_d}(\theta) = \overline{R_{M_d}(\theta)} \setminus R_{M_d}(\theta) \subseteq \partial M_d$ is a continuum and in particular connected. Let $c \in Cl_{M_d}(\theta)$. By Propositions 1.2 and 1.4, c has either parabolic or Misiurewicz dynamics. Since the period q is fixed, this implies that c varies in a finite set and is thus unique, which proves convergence. The dichotomy follows from Proposition 1.2 and Proposition 1.4. The rest of the statements 1. and 2. follow by combining Proposition 1.2, Proposition 1.4 and Proposition 1.6. **q.e.d.**

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